Relative M-groups and relative Mp-groups

陈晓友 (河南工业大学)

Xiaoyou Chen (Henan University of Technology)

In Honor Of Prof. Broué

Southern University of Science and Technology

May 27, 2026

M-groups

- Let G be a finite group and $\mathbb{C}$ be the set of complex numbers.

M-groups

- Let G be a finite group and $\mathbb{C}$ be the set of complex numbers.
- Let $\mathbb{C}$-representation $\mathfrak{X} : G \to \mathrm{GL}(n, \mathbb{C}), g \mapsto \mathfrak{X}(g)$ afford character $\chi : G \to \mathbb{C}, g \mapsto \chi(g) = \mathrm{tr}(\mathfrak{X}(g))$. $\chi(1) = n$ is the degree of χ.

M-groups

- Let G be a finite group and $\mathbb{C}$ be the set of complex numbers.
- Let $\mathbb{C}$-representation $\mathfrak{X} : G \to \mathrm{GL}(n, \mathbb{C}), g \mapsto \mathfrak{X}(g)$ afford character $\chi : G \to \mathbb{C}, g \mapsto \chi(g) = \mathrm{tr}(\mathfrak{X}(g))$. $\chi(1) = n$ is the degree of χ.
- Denote by $\mathrm{Irr}(G)$ the set of irreducible (complex) characters of G.

Definiton of induced character

Let $H \subseteq G$ and $\varphi \in \mathrm{Char}(H)$. Then φ^G, the *induced character* on G, is given by

$$\varphi^G(g) = \frac{1}{|H|} \sum_{x \in G} \varphi^0(xgx^{-1}),$$

where φ^0 is defined by

$$\varphi^0(y) = \begin{cases} \varphi(y) & \text{if } y \in H \\ 0 & \text{if } y \notin H. \end{cases}$$

M-groups

- Let G be a finite group and $\mathbb{C}$ be the set of complex numbers.
- Let $\mathbb{C}$-representation $\mathfrak{X} : G \to \mathrm{GL}(n, \mathbb{C}), g \mapsto \mathfrak{X}(g)$ afford character $\chi : G \to \mathbb{C}, g \mapsto \chi(g) = \mathrm{tr}(\mathfrak{X}(g))$. $\chi(1) = n$ is the degree of χ.
- Denote by $\mathrm{Irr}(G)$ the set of irreducible (complex) characters of G.

Definiton of induced character

Let $H \subseteq G$ and $\varphi \in \mathrm{Char}(H)$. Then φ^G, the *induced character* on G, is given by

$$\varphi^G(g) = \frac{1}{|H|} \sum_{x \in G} \varphi^0(xgx^{-1}),$$

where φ^0 is defined by

$$\varphi^0(y) = \begin{cases} \varphi(y) & \text{if } y \in H \\ 0 & \text{if } y \notin H. \end{cases}$$

M-groups

Definiton of An M-group[1, Corollary 5.10]

A character of G is monomial if it is induced from a linear character of some (not necessarily proper) subgroup of G. The group G is an M-group if every $\chi \in \mathrm{Irr}(G)$ is monomial.

Taketa's Theorem[1, Corollary 5.13]

An M-group must be solvable.

I. M. Isaacs, *Character Theory of Finite Groups*, Academic Press, New York, 1976.

M-groups

Definiton of An M-group[1, Corollary 5.10]

A character of G is monomial if it is induced from a linear character of some (not necessarily proper) subgroup of G. The group G is an M-group if every $\chi \in \mathrm{Irr}(G)$ is monomial.

Taketa's Theorem[1, Corollary 5.13]

An M-group must be solvable.

I. M. Isaacs, *Character Theory of Finite Groups*, Academic Press, New York, 1976.

M-groups

- A solvable group may not be an M-group

Character Table of $\mathrm{SL}(2,3)$

	1a	6a	6b	2a	3a	3b	4a
χ_1	1	1	1	1	1	1	1
χ_2	1	ω	$\bar{\omega}$	1	$\bar{\omega}$	ω	1
χ_3	1	$\bar{\omega}$	ω	1	ω	$\bar{\omega}$	1
χ_4	2	1	1	-2	-1	-1	0
χ_5	2	$\bar{\omega}$	ω	-2	$-\omega$	$-\bar{\omega}$	0
χ_6	2	ω	$\bar{\omega}$	-2	$-\bar{\omega}$	$-\omega$	0
χ_7	3	0	0	3	0	0	-1

where $\omega = E^3 = (-1-\sqrt{3}i)/2 = -1 - b3$

M-groups

- A solvable group may not be an M-group

Character Table of $\mathrm{SL}(2,3)$

	1a	6a	6b	2a	3a	3b	4a
χ_1	1	1	1	1	1	1	1
χ_2	1	ω	$\bar{\omega}$	1	$\bar{\omega}$	ω	1
χ_3	1	$\bar{\omega}$	ω	1	ω	$\bar{\omega}$	1
χ_4	2	1	1	-2	-1	-1	0
χ_5	2	$\bar{\omega}$	ω	-2	$-\omega$	$-\bar{\omega}$	0
χ_6	2	ω	$\bar{\omega}$	-2	$-\bar{\omega}$	$-\omega$	0
χ_7	3	0	0	3	0	0	-1

where $\omega = E^3 = (-1-\sqrt{3}i)/2 = -1-b3$

M-groups

Dornhoff's Theorem

Let G be an M-group and N be a normal Hall subgroup of G. Then N is also an M-group.

M-groups

Dornhoff's Theorem

Let G be an M-group and N be a normal Hall subgroup of G. Then N is also an M-group.

L. Dornhoff, M-groups and 2-groups, Math. Z. **100** (1967), 226-256.

M-groups

Dornhoff's Theorem

Let G be an M-group and N be a normal Hall subgroup of G. Then N is also an M-group.

L. Dornhoff, M-groups and 2-groups, Math. Z. **100** (1967), 226-256.

M-groups

Dornhoff's Theorem

Let G be an M-group and N be a normal Hall subgroup of G. Then N is also an M-group.

L. Dornhoff, M-groups and 2-groups, Math. Z. **100** (1967), 226-256.

Question

Can the condition 'normal Hall' be weakened?

M-groups

Dornhoff's Theorem

Let G be an M-group and N be a normal Hall subgroup of G. Then N is also an M-group.

L. Dornhoff, M-groups and 2-groups, Math. Z. **100** (1967), 226-256.

Question

Can the condition 'normal Hall' be weakened?

M-groups

Dornhoff's Theorem

Let G be an M-group and N be a normal Hall subgroup of G. Then N is also an M-group.

L. Dornhoff, M-groups and 2-groups, Math. Z. **100** (1967), 226-256.

Question

Can the condition 'normal Hall' be weakened?

E. C. Dade, Normal subgroups of M-groups need not be M-groups, Math. Z. **133** (1973), 313-317.

M-groups

Dornhoff's Theorem

Let G be an M-group and N be a normal Hall subgroup of G. Then N is also an M-group.

L. Dornhoff, M-groups and 2-groups, Math. Z. **100** (1967), 226-256.

Question

Can the condition 'normal Hall' be weakened?

E. C. Dade, Normal subgroups of M-groups need not be M-groups, Math. Z. **133** (1973), 313-317.

R. W. van der Waall, On the embedding of minimal non-M-groups, Indag. Math. **77** (1974), 157-167.

M-groups

Dornhoff's Theorem

Let G be an M-group and N be a normal Hall subgroup of G. Then N is also an M-group.

L. Dornhoff, M-groups and 2-groups, Math. Z. **100** (1967), 226-256.

Question

Can the condition 'normal Hall' be weakened?

E. C. Dade, Normal subgroups of M-groups need not be M-groups, Math. Z. **133** (1973), 313-317.

R. W. van der Waall, On the embedding of minimal non-M-groups, Indag. Math. **77** (1974), 157-167.

H. Fukushima, Hall subgroups of M-groups need not be M-groups, Proc. Amer. Math. Soc., **133** (2004), 671-675.

M-groups

Dornhoff's Theorem

Let G be an M-group and N be a normal Hall subgroup of G. Then N is also an M-group.

L. Dornhoff, M-groups and 2-groups, Math. Z. **100** (1967), 226-256.

Question

Can the condition 'normal Hall' be weakened?

E. C. Dade, Normal subgroups of M-groups need not be M-groups, Math. Z. **133** (1973), 313-317.

R. W. van der Waall, On the embedding of minimal non-M-groups, Indag. Math. **77** (1974), 157-167.

H. Fukushima, Hall subgroups of M-groups need not be M-groups, Proc. Amer. Math. Soc., **133** (2004), 671-675.

Relative M-groups

I. M. Isaacs, *Character Theory of Finite Groups*, Academic Press, New York, 1976.

Definiton of a Relative M-group

Let $N \lhd G$ and $\chi \in \mathrm{Irr}(G)$. Then χ is a relative M-character with respect to N if there exists H with $N \subseteq H \subseteq G$ and $\psi \in \mathrm{Irr}(H)$ such that $\psi^G = \chi$ and ψ_N is irreducible.
If every $\chi \in \mathrm{Irr}(G)$ is a relative M-character with respect to N, then G is a relative M-group with respect to N.

Properties

If G is a relative M-group with respect to N, then G/N is an M-group.
However, if G/N is an M-group, then we do not necessarily have that G is a relative M-group with respect to N.

Relative M-groups

Example

$\mathrm{GL}(2,3)$ is a relative M-group with respect to $\mathrm{SL}(2,3)$.

Relative M-groups

Example

$\mathrm{GL}(2,3)$ is a relative M-group with respect to $\mathrm{SL}(2,3)$.

Example

S_5 is a relative M-group with respect to A_5, however, S_5 is not solvable.

Relative M-groups

Example

$\mathrm{GL}(2,3)$ is a relative M-group with respect to $\mathrm{SL}(2,3)$.

Example

S_5 is a relative M-group with respect to A_5, however, S_5 is not solvable.

Example

$\mathrm{SL}(2,3)/\mathbf{Z}(\mathrm{SL}(2,3)) \cong A_4$ is an M-group, however, $\mathrm{SL}(2,3)$ is not a relative M-group with respect to $\mathbf{Z}(\mathrm{SL}(2,3))$.

Relative M-groups

Example

$\mathrm{GL}(2,3)$ is a relative M-group with respect to $\mathrm{SL}(2,3)$.

Example

S_5 is a relative M-group with respect to A_5, however, S_5 is not solvable.

Example

$\mathrm{SL}(2,3)/\mathbf{Z}(\mathrm{SL}(2,3)) \cong A_4$ is an M-group, however, $\mathrm{SL}(2,3)$ is not a relative M-group with respect to $\mathbf{Z}(\mathrm{SL}(2,3))$.

Kwang-Wu Chen, On relative M-groups, Chinese J. Math. **21** (1993), 1-12.

Relative M-groups

Example

$\mathrm{GL}(2,3)$ is a relative M-group with respect to $\mathrm{SL}(2,3)$.

Example

S_5 is a relative M-group with respect to A_5, however, S_5 is not solvable.

Example

$\mathrm{SL}(2,3)/\mathbf{Z}(\mathrm{SL}(2,3)) \cong A_4$ is an M-group, however, $\mathrm{SL}(2,3)$ is not a relative M-group with respect to $\mathbf{Z}(\mathrm{SL}(2,3))$.

Kwang-Wu Chen, On relative M-groups, Chinese J. Math. **21** (1993), 1-12.

Relative M-groups

Character Table of $\mathrm{SL}(2,3)$

	1a	6a	6b	2a	3a	3b	4a
χ_1	1	1	1	1	1	1	1
χ_2	1	ω	$\bar{\omega}$	1	$\bar{\omega}$	ω	1
χ_3	1	$\bar{\omega}$	ω	1	ω	$\bar{\omega}$	1
χ_4	2	1	1	-2	-1	-1	0
χ_5	2	$\bar{\omega}$	ω	-2	$-\omega$	$-\bar{\omega}$	0
χ_6	2	ω	$\bar{\omega}$	-2	$-\bar{\omega}$	$-\omega$	0
χ_7	3	0	0	3	0	0	-1

where $\omega = E^3 = (-1-\sqrt{3}i)/2 = -1 - b3$

Relative M-groups

Character Table of $\mathrm{SL}(2,3)$

	1a	6a	6b	2a	3a	3b	4a
χ_1	1	1	1	1	1	1	1
χ_2	1	ω	$\bar{\omega}$	1	$\bar{\omega}$	ω	1
χ_3	1	$\bar{\omega}$	ω	1	ω	$\bar{\omega}$	1
χ_4	2	1	1	-2	-1	-1	0
χ_5	2	$\bar{\omega}$	ω	-2	$-\omega$	$-\bar{\omega}$	0
χ_6	2	ω	$\bar{\omega}$	-2	$-\bar{\omega}$	$-\omega$	0
χ_7	3	0	0	3	0	0	-1

where $\omega = E^3 = (-1-\sqrt{3}i)/2 = -1 - b3$

Relative M-groups

Character Table of S_5

	1a	2a	3a	5a	2b	4a	6a
χ_1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1
χ_3	6	-2	0	1	0	0	0
χ_4	4	0	1	-1	2	0	-1
χ_5	4	0	1	-1	-2	0	1
χ_6	5	1	-1	0	1	-1	1
χ_7	5	1	-1	0	-1	1	-1

Relative M-groups

Character Table of S_5

	1a	2a	3a	5a	2b	4a	6a
χ_1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1
χ_3	6	-2	0	1	0	0	0
χ_4	4	0	1	-1	2	0	-1
χ_5	4	0	1	-1	-2	0	1
χ_6	5	1	-1	0	1	-1	1
χ_7	5	1	-1	0	-1	1	-1

Relative M-groups

Character Table of A_5

	1a	2a	3a	5a	5b
χ_1	1	1	1	1	1
χ_2	3	-1	0	A	$*A$
χ_3	3	-1	0	$*A$	A
χ_4	4	0	1	-1	-1
χ_5	5	1	-1	0	0

where $A = (1-\sqrt{5})/2 = -b5$ and $*A = (1+\sqrt{5})/2$.

Brauer characters

G. Navarro, *Characters and Blocks of Finite Groups*, Cambridge University Press, Cambridge, 1998.

Brauer characters

G. Navarro, *Characters and Blocks of Finite Groups*, Cambridge University Press, Cambridge, 1998.

- Let F be an algebraically closed field over $\mathbb{Z}_p$.

Brauer characters

G. Navarro, *Characters and Blocks of Finite Groups*, Cambridge University Press, Cambridge, 1998.

- Let F be an algebraically closed field over $\mathbb{Z}_p$.
- Let $\mathcal{X} : G \to \mathrm{GL}(n, F)$ be an F-representation of G.

Brauer characters

G. Navarro, *Characters and Blocks of Finite Groups*, Cambridge University Press, Cambridge, 1998.

- Let F be an algebraically closed field over $\mathbb{Z}_p$.
- Let $\mathcal{X} : G \to \mathrm{GL}(n, F)$ be an F-representation of G.
- Let $g \in G^0$. Then there exist uniquely determined $\xi_1, \ldots, \xi_n \in \mathbf{U}$ such that $\xi_1^*, \ldots, \xi_n^*$ are eigenvalues of $\mathcal{X}(g)$. For $g \in G^0$, we say that
$$\varphi : G^0 \to \mathbb{C}$$
by $\varphi(g) = \xi_1 + \cdots + \xi_n$ is a Brauer character of G afforded by $\mathcal{X}$.

Brauer characters

G. Navarro, *Characters and Blocks of Finite Groups*, Cambridge University Press, Cambridge, 1998.

- Let F be an algebraically closed field over $\mathbb{Z}_p$.
- Let $\mathcal{X} : G \to \mathrm{GL}(n, F)$ be an F-representation of G.
- Let $g \in G^0$. Then there exist uniquely determined $\xi_1, \ldots, \xi_n \in \mathbf{U}$ such that $\xi_1^*, \ldots, \xi_n^*$ are eigenvalues of $\mathcal{X}(g)$. For $g \in G^0$, we say that
$$\varphi : G^0 \to \mathbb{C}$$
by $\varphi(g) = \xi_1 + \cdots + \xi_n$ is a Brauer character of G afforded by $\mathcal{X}$.
- φ is irreducible if $\mathcal{X}$ is irreducible. φ is nonlinear if $\varphi(1) > 1$. Write $\mathrm{IBr}(G)$ for the set of irreducible $(p-)$Brauer characters of G.

Brauer characters

G. Navarro, *Characters and Blocks of Finite Groups*, Cambridge University Press, Cambridge, 1998.

- Let F be an algebraically closed field over $\mathbb{Z}_p$.
- Let $\mathcal{X} : G \to \mathrm{GL}(n, F)$ be an F-representation of G.
- Let $g \in G^0$. Then there exist uniquely determined $\xi_1, \ldots, \xi_n \in \mathbf{U}$ such that $\xi_1^*, \ldots, \xi_n^*$ are eigenvalues of $\mathcal{X}(g)$. For $g \in G^0$, we say that
$$\varphi : G^0 \to \mathbb{C}$$
by $\varphi(g) = \xi_1 + \cdots + \xi_n$ is a Brauer character of G afforded by $\mathcal{X}$.
- φ is irreducible if $\mathcal{X}$ is irreducible. φ is nonlinear if $\varphi(1) > 1$. Write $\mathrm{IBr}(G)$ for the set of irreducible $(p-)$Brauer characters of G.

M_p-groups

Definiton of an M_p-group

Let G be a finite group and p be a prime. Then G is called an M_p-group if every $\varphi \in \mathrm{IBr}(G)$ is monomial.

M_p-groups

Definiton of an M_p-group

Let G be a finite group and p be a prime. Then G is called an M_p-group if every $\varphi \in \mathrm{IBr}(G)$ is monomial.

Okuyama's Theorem

Let G be a finite group, p be a prime. If G is an M_p-group, then G is solvable.

M_p-groups

Definiton of an M_p-group

Let G be a finite group and p be a prime. Then G is called an M_p-group if every $\varphi \in \mathrm{IBr}(G)$ is monomial.

Okuyama's Theorem

Let G be a finite group, p be a prime. If G is an M_p-group, then G is solvable.

- An M-group is an M_p-group.
- An M_p-group may not be an M-group; for example, $SL(2,3)$ is an M_2-group, however, it is not an M-group.

M_p-groups

Definiton of an M_p-group

Let G be a finite group and p be a prime. Then G is called an M_p-group if every $\varphi \in \mathrm{IBr}(G)$ is monomial.

Okuyama's Theorem

Let G be a finite group, p be a prime. If G is an M_p-group, then G is solvable.

- An M-group is an M_p-group.
- An M_p-group may not be an M-group; for example, $SL(2,3)$ is an M_2-group, however, it is not an M-group.

M_p-groups

Definiton of an M_p-group

Let G be a finite group and p be a prime. Then G is called an M_p-group if every $\varphi \in \mathrm{IBr}(G)$ is monomial.

Okuyama's Theorem

Let G be a finite group, p be a prime. If G is an M_p-group, then G is solvable.

- An M-group is an M_p-group.
- An M_p-group may not be an M-group; for example, $SL(2,3)$ is an M_2-group, however, it is not an M-group.

T. Okuyama, Module correspondence in finite groups, Hokkaido Math. J., **10** (1981), 299-318.

M_p-groups

Definiton of an M_p-group

Let G be a finite group and p be a prime. Then G is called an M_p-group if every $\varphi \in \mathrm{IBr}(G)$ is monomial.

Okuyama's Theorem

Let G be a finite group, p be a prime. If G is an M_p-group, then G is solvable.

- An M-group is an M_p-group.
- An M_p-group may not be an M-group; for example, $SL(2,3)$ is an M_2-group, however, it is not an M-group.

T. Okuyama, Module correspondence in finite groups, Hokkaido Math. J., **10** (1981), 299-318.

M_p-groups

Bessenrodt's Theorem

Let G be a finite group, p be a prime. Assume that G is an M_p-group and N is a normal Hall subgroup of G. Then N is also an M_p-group.

M_p-groups

Bessenrodt's Theorem

Let G be a finite group, p be a prime. Assume that G is an M_p-group and N is a normal Hall subgroup of G. Then N is also an M_p-group.

M_p-groups

Bessenrodt's Theorem

Let G be a finite group, p be a prime. Assume that G is an M_p-group and N is a normal Hall subgroup of G. Then N is also an M_p-group.

C. Bessenrodt, Monomial representations and generalizations, J. Austral. Math. Soc. Ser. A **48** (1990), 264-280.

M_p-groups

Bessenrodt's Theorem

Let G be a finite group, p be a prime. Assume that G is an M_p-group and N is a normal Hall subgroup of G. Then N is also an M_p-group.

C. Bessenrodt, Monomial representations and generalizations, J. Austral. Math. Soc. Ser. A **48** (1990), 264-280.

Relative M_p-groups

Definiton

Let $N \lhd G$, p be a prime and $\varphi \in \mathrm{IBr}(G)$. The φ is a relative M_p-character with respect to N if there exists H with $N \subseteq H \subseteq G$ and $\theta \in \mathrm{IBr}(H)$ such that $\theta^G = \varphi$ and $\theta_N \in \mathrm{IBr}(N)$.
If every $\varphi \in \mathrm{IBr}(G)$ is a relative M_p-character with respect to N, then G is a relative M_p-group with respect to N.

Property

If G is a relative M_p-group with respect to N, then G/N is an M_p-group. However, if G/N is an M_p-group, then we do not necessarily have that G is a relative M_p-group with respect to N.

Relative M_p-groups

Example

S_5 is a relative M_2-group with respect to A_5, however, S_5 is not solvable.

Relative M_p-groups

Example

S_5 is a relative M_2-group with respect to A_5, however, S_5 is not solvable.

Example

$SL(2,3)/\mathbf{Z}(SL(2,3)) \cong A_4$ is an M_3-group, however, $SL(2,3)$ is not a relative M_3-group with respect to $\mathbf{Z}(SL(2,3))$.

Relative M_p-groups

Example

S_5 is a relative M_2-group with respect to A_5, however, S_5 is not solvable.

Example

$SL(2,3)/\mathbf{Z}(SL(2,3)) \cong A_4$ is an M_3-group, however, $SL(2,3)$ is not a relative M_3-group with respect to $\mathbf{Z}(SL(2,3))$.

Relative M_p-groups

Relative M_p-groups

Theorem

Let G be a group, $N \lhd G$ and p be a prime. Assume that G is a relative M_p-group with respect to N and H is a normal Hall subgroup of G containing N. Then H is also a relative M_p-group with respect to N.

Relative M_p-groups

Theorem

Let G be a group, $N \lhd G$ and p be a prime. Assume that G is a relative M_p-group with respect to N and H is a normal Hall subgroup of G containing N. Then H is also a relative M_p-group with respect to N.

Corollary

In the above theorem, if we take $N = 1$, then we have that a normal Hall subgroup of an M_p-group is also an M_p-group.

Relative M_p-groups

Theorem

Let G be a group, $N \lhd G$ and p be a prime. Assume that G is a relative M_p-group with respect to N and H is a normal Hall subgroup of G containing N. Then H is also a relative M_p-group with respect to N.

Corollary

In the above theorem, if we take $N = 1$, then we have that a normal Hall subgroup of an M_p-group is also an M_p-group.

Xiaoyou Chen and Ni Du, A generalization on Bessenrodt's corollary, Comm. Algebra, **53** (2025), 2299-2301.

Relative M_p-groups

Theorem

Let G be a group, $N \lhd G$ and p be a prime. Assume that G is a relative M_p-group with respect to N and H is a normal Hall subgroup of G containing N. Then H is also a relative M_p-group with respect to N.

Corollary

In the above theorem, if we take $N = 1$, then we have that a normal Hall subgroup of an M_p-group is also an M_p-group.

Xiaoyou Chen and Ni Du, A generalization on Bessenrodt's corollary, Comm. Algebra, **53** (2025), 2299-2301.

Questions

Questions

Question

Let G be a relative M-group with respect to N. Can we have that G is a relative M_p-group with respect to N for every prime p?

Questions

Question

Let G be a relative M-group with respect to N. Can we have that G is a relative M_p-group with respect to N for every prime p?

Example

S_5 is a relative M-group with respect to A_5, and S_5 is a relative M_p-group with respect to A_5, for every prime p.

Questions

Question

Let G be a relative M-group with respect to N. Can we have that G is a relative M_p-group with respect to N for every prime p?

Example

S_5 is a relative M-group with respect to A_5, and S_5 is a relative M_p-group with respect to A_5, for every prime p.

Relative M_p-groups

Relative M_p-groups

Theorem

Let G be a group and N be an abelian normal subgroup of G. If G is an M_p-group, then G is a relative M_p-group with respect to N.

Relative M_p-groups

Theorem

Let G be a group and N be an abelian normal subgroup of G. If G is an M_p-group, then G is a relative M_p-group with respect to N.

Corollary

Let G be a group and N be an abelian normal subgroup of G. If G is an M-group, then G is a relative M-group with respect to N.

Relative M_p-groups

Theorem

Let G be a group and N be an abelian normal subgroup of G. If G is an M_p-group, then G is a relative M_p-group with respect to N.

Corollary

Let G be a group and N be an abelian normal subgroup of G. If G is an M-group, then G is a relative M-group with respect to N.

Theorem

Let G be a relative M_p-group with respect to a normal subgroup N. Let $A \lhd G$ and $A \leq N$. Then G/A is a relative M_p-group with respect to N/A.

Relative M_p-groups

Theorem

Let G be a group and N be an abelian normal subgroup of G. If G is an M_p-group, then G is a relative M_p-group with respect to N.

Corollary

Let G be a group and N be an abelian normal subgroup of G. If G is an M-group, then G is a relative M-group with respect to N.

Theorem

Let G be a relative M_p-group with respect to a normal subgroup N. Let $A \lhd G$ and $A \leq N$. Then G/A is a relative M_p-group with respect to N/A.

Questions

Questions

M. Broué and L. Puig, A Frobenius theorem for blocks, Invent. Math., **56** (1980), 117-128.

Questions

M. Broué and L. Puig, A Frobenius theorem for blocks, Invent. Math., **56** (1980), 117-128.

G. Navarro, Nilpotent characters, Pacific J. Math., **169** (1995), 343-351.

Questions

M. Broué and L. Puig, A Frobenius theorem for blocks, Invent. Math., **56** (1980), 117-128.

G. Navarro, Nilpotent characters, Pacific J. Math., **169** (1995), 343-351.

Questions

- M. Broué and L. Puig, A Frobenius theorem for blocks, Invent. Math., **56** (1980), 117-128.
- G. Navarro, Nilpotent characters, Pacific J. Math., **169** (1995), 343-351.
- C. Bessenrodt, Monomial representations and generalizations, J. Austral. Math. Soc. Ser. A, **48** (1990), 264-280.

- If a p-block B is a nilpotent block, then $|\mathrm{IBr}(B)| = 1$, where $\mathrm{IBr}(B) = B \cap \mathrm{IBr}(G)$.

Questions

M. Broué and L. Puig, A Frobenius theorem for blocks, Invent. Math., **56** (1980), 117-128.

G. Navarro, Nilpotent characters, Pacific J. Math., **169** (1995), 343-351.

C. Bessenrodt, Monomial representations and generalizations, J. Austral. Math. Soc. Ser. A, **48** (1990), 264-280.

- If a p-block B is a nilpotent block, then $|\mathrm{IBr}(B)| = 1$, where $\mathrm{IBr}(B) = B \cap \mathrm{IBr}(G)$.
- Let B be a p-block of p-solvable group G. Then B is a p-block if and only if it contains a nilpotent Brauer character.
- A p-block B of a group is said to be an M-block if all the (complex) irreducible characters in B are monomial.

Questions

M. Broué and L. Puig, A Frobenius theorem for blocks, Invent. Math., **56** (1980), 117-128.

G. Navarro, Nilpotent characters, Pacific J. Math., **169** (1995), 343-351.

C. Bessenrodt, Monomial representations and generalizations, J. Austral. Math. Soc. Ser. A, **48** (1990), 264-280.

- If a p-block B is a nilpotent block, then $|\mathrm{IBr}(B)| = 1$, where $\mathrm{IBr}(B) = B \cap \mathrm{IBr}(G)$.
- Let B be a p-block of p-solvable group G. Then B is a p-block if and only if it contains a nilpotent Brauer character.
- A p-block B of a group is said to be an M-block if all the (complex) irreducible characters in B are monomial.

Questions

Example

$\mathrm{SL}(2,3)$ has three 3-blocks: B_0, B_1, B_2; the principal block B_0, B_2 with defect zero are M-blocks which are also nilpotent block; however, block B_1 with defect 1 which is a nilpotent block is not an M-block.

Questions

Example

$\mathrm{SL}(2,3)$ has three 3-blocks: B_0, B_1, B_2; the principal block B_0, B_2 with defect zero are M-blocks which are also nilpotent block; however, block B_1 with defect 1 which is a nilpotent block is not an M-block.

Example

S_3 has one 3-block: the principal block, which is an M-block, is not a nilpotent block.

Questions

Example

$\mathrm{SL}(2,3)$ has three 3-blocks: B_0, B_1, B_2; the principal block B_0, B_2 with defect zero are M-blocks which are also nilpotent block; however, block B_1 with defect 1 which is a nilpotent block is not an M-block.

Example

S_3 has one 3-block: the principal block, which is an M-block, is not a nilpotent block.

Question

When can an M-block be a nilpotent block?

Questions

Example

S_5 is a relative M-group with respect to A_5. S_5 has one 3-block with defect zero which is an M-block.

Questions

Example

S_5 is a relative M-group with respect to A_5. S_5 has one 3-block with defect zero which is an M-block.

- A p-block B of a group is said to be a relative M-block if all the characters in $\mathrm{Irr}(B)(= \mathrm{Irr}(G) \cap B)$ are relative with respect to N.

Questions

Example

S_5 is a relative M-group with respect to A_5. S_5 has one 3-block with defect zero which is an M-block.

- A p-block B of a group is said to be a relative M-block if all the characters in $\mathrm{Irr}(B)(= \mathrm{Irr}(G) \cap B)$ are relative with respect to N.

Question

When can a relative M-block be a M-block?

Questions

Example

S_5 is a relative M-group with respect to A_5. S_5 has one 3-block with defect zero which is an M-block.

- A p-block B of a group is said to be a relative M-block if all the characters in $\mathrm{Irr}(B)(= \mathrm{Irr}(G) \cap B)$ are relative with respect to N.

Question

When can a relative M-block be a M-block?

Acknowledgements

- Thank Conference Committee and thank everyone for your attention!

Thank you!